# Neighbour balance and evenness of distribution designs for use in field variety trials

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Blocking is a standard strategy for improving the precision of randomized experimental designs.

In field trials, it is usually beneficial to impose blocking in both the row and column directions.

Here is an example:

# A row-column design for 34 treatments with 9 rows and 13 columns

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 24 | 18 | 3 | 25 | 9 | 32 | 20 | 15 | 19 | 17 | 26 | 2 | 34 |
| 34 | 27 | 16 | 14 | 25 | 6 | 15 | 21 | 11 | 24 | 30 | 12 | 9 |
| 30 | 20 | 2 | 28 | 13 | 21 | 7 | 4 | 14 | 34 | 33 | 32 | 8 |
| 5 | 11 | 17 | 33 | 21 | 29 | 25 | 34 | 13 | 23 | 3 | 27 | 26 |
| 7 | 29 | 30 | 24 | 28 | 26 | 23 | 31 | 10 | 4 | 15 | 22 | 11 |
| 29 | 32 | 31 | 20 | 22 | 9 | 5 | 27 | 30 | 10 | 8 | 17 | 1 |
| 19 | 26 | 21 | 1 | 10 | 33 | 16 | 22 | 6 | 25 | 20 | 7 | 2 |
| 33 | 16 | 24 | 27 | 18 | 31 | 13 | 19 | 32 | 8 | 12 | 23 | 28 |
| 18 | 4 | 28 | 29 | 12 | 23 | 31 | 1 | 3 | 6 | 22 | 14 | 5 |

A common problem with classical randomized row-column designs is that replications of a treatment may be clustered and some pairs of treatments may appear next to each other more often than others.

Such patterns adversely affect certain design properties which we refer to as **neighbour balance** (**NB**) and **evenness of distribution** (**ED**), and they may be particularly harmful if some of the treatments are very susceptible to environmental stresses.  Here is the design again:

# A row-column design for 34 treatments with 9 rows and 13 columns

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 24 | 18 | 3 | 25 | 9 | 32 | 20 | 15 | 19 | 17 | 26 | 2 | 34 |
| 34 | 27 | 16 | 14 | 25 | 6 | 15 | 21 | 11 | 24 | 30 | 12 | 9 |
| 30 | 20 | 2 | 28 | 13 | 21 | 7 | 4 | 14 | 34 | 33 | 32 | 8 |
| 5 | 11 | 17 | 33 | 21 | 29 | 25 | 34 | 13 | 23 | 3 | 27 | 26 |
| 7 | 29 | 30 | 24 | 28 | 26 | 23 | 31 | 10 | 4 | 15 | 22 | 11 |
| 29 | 32 | 31 | 20 | 22 | 9 | 5 | 27 | 30 | 10 | 8 | 17 | 1 |
| 19 | 26 | 21 | 1 | 10 | 33 | 16 | 22 | 6 | 25 | 20 | 7 | 2 |
| 33 | 16 | 24 | 27 | 18 | 31 | 13 | 19 | 32 | 8 | 12 | 23 | 28 |
| 18 | 4 | 28 | 29 | 12 | 23 | 31 | 1 | 3 | 6 | 22 | 14 | 5 |

The design has poor evenness of distribution (ED) with replications of treatment 21 clustered closely together in the middle columns. Treatment 8 only spans 4 of the columns and treatment 10 spans just 3 of the rows.  Additionally, there are multiple instances of diagonal self-adjacencies: treatments 10, 15, 21, 25, 29 and 31.

We also have poor neighbour balance (NB) e.g. treatments 8 and 32 are adjacent in rows 3 and 8 also look on the pairs (24,30) and (12,23).

To address these problems, an NB&ED design can be generated using CycDesigN (https://vsni.co.uk/software/cycdesign)

# An NB&ED design generated by CycDesigN

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 25 | 32 | 14 | 22 | 29 | 4 | 19 | 21 | 26 | 11 | 24 | 17 | 28 |
| 21 | 23 | 8 | 1 | 24 | 33 | 25 | 30 | 31 | 22 | 10 | 27 | 5 |
| 7 | 29 | 2 | 31 | 21 | 27 | 12 | 20 | 16 | 9 | 33 | 15 | 34 |
| 24 | 20 | 11 | 30 | 28 | 34 | 14 | 6 | 18 | 3 | 13 | 32 | 26 |
| 28 | 9 | 27 | 6 | 25 | 26 | 13 | 17 | 2 | 7 | 19 | 12 | 22 |
| 31 | 4 | 33 | 16 | 18 | 11 | 10 | 28 | 15 | 8 | 14 | 21 | 1 |
| 30 | 3 | 5 | 19 | 20 | 32 | 1 | 33 | 27 | 34 | 29 | 31 | 23 |
| 16 | 34 | 17 | 12 | 23 | 7 | 22 | 24 | 5 | 25 | 4 | 20 | 8 |
| 26 | 18 | 13 | 10 | 15 | 9 | 3 | 29 | 32 | 23 | 30 | 6 | 2 |

The minimum span is now 8 columns and 6 rows, compared to only 4 columns and 3 rows for the previous design.

Here we have a slight reduction in the average efficiency factor (0.83, compared to 0.85 before) but this is compensated for by the improved neighbour balance and evenness of distribution.  For larger designs any drop in efficiency is likely to be much smaller than this, if at all.

When possible, resolvable designs are preferred, and in particular Latinized row-column designs. Even though the replicates provide some protection against evenness of distribution, it still needs to be considered and so does neighbour balance.  Here is an example of a non-spatial Latinized row-column design produced by CycDesigN with the replicates in a 2 x 2 array.

# A non-spatial Latinized row-column design for 24 treatments and 2 x 2 replicates

| | | Column | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 1 | 2 | 3 | 4 | 5 | 6 | |
| | Row | | | | | | | | | | | | | |
| | 1 | 3 | 1 | 12 | 11 | 6 | 8 | 23 | 18 | 13 | 14 | 22 | 20 | |
| | 2 | 10 | 24 | 9 | 5 | 23 | 18 | 19 | 2 | 8 | 6 | 17 | 4 | |
| Replicate 1 | 3 | 17 | 13 | 20 | 19 | 15 | 7 | 21 | 1 | 16 | 5 | 12 | 10 | Replicate 2 |
| | 4 | 14 | 4 | 2 | 22 | 16 | 21 | 3 | 15 | 9 | 7 | 24 | 11 | |
| | 1 | 13 | 11 | 15 | 14 | 21 | 12 | 22 | 20 | 7 | 16 | 1 | 3 | |
| Replicate 3 | 2 | 22 | 6 | 4 | 18 | 9 | 3 | 8 | 24 | 23 | 11 | 14 | 2 | Replicate 4 |
| | 3 | 1 | 23 | 8 | 20 | 17 | 5 | 10 | 6 | 12 | 18 | 19 | 13 | |
| | 4 | 7 | 2 | 19 | 24 | 10 | 16 | 15 | 21 | 4 | 17 | 9 | 5 | |

In this example treatment 21 has replicates diagonally adjacent to each other, as does treatment 7. These are examples of treatment clumping and it is something that the use of resolvable designs helps to minimize. In this instance the issue is not too severe; nevertheless users often feel more comfortable if the layout can avoid any such juxtapositions. Even the existence of knights move proximities can raise eyebrows; e.g. treatments 18 and 23 in replicates 1 and 2. There are also multiple instances of pairs of treatments as neighbours more than once; e.g. treatment pairs 10 and 24 in the row direction in replicates 1 and 2.

NB&ED designs aim to avoid these issues as can be seen in the next slide.

# A spatial Latinized row-column design for 24 treatments and 2 x 2 replicates

| | | Column | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Row | 1 | 2 | 3 | 4 | 5 | 6 | 1 | 2 | 3 | 4 | 5 | 6 | |
| | 1 | 1 | 14 | 15 | 21 | 8 | 2 | 18 | 16 | 20 | 19 | 17 | 11 | |
| | 2 | 23 | 6 | 13 | 12 | 22 | 17 | 3 | 10 | 15 | 2 | 9 | 7 | |
| Replicate 1 | 3 | 3 | 20 | 24 | 10 | 11 | 19 | 4 | 8 | 5 | 23 | 14 | 12 | Replicate 2 |
| | 4 | 16 | 7 | 4 | 18 | 9 | 5 | 21 | 6 | 22 | 24 | 13 | 1 | |
| | 1 | 21 | 13 | 22 | 16 | 2 | 7 | 1 | 9 | 17 | 10 | 6 | 18 | |
| Replicate 3 | 2 | 5 | 18 | 1 | 24 | 12 | 6 | 19 | 23 | 7 | 15 | 4 | 22 | Replicate 4 |
| | 3 | 11 | 23 | 10 | 14 | 17 | 15 | 13 | 20 | 8 | 12 | 2 | 3 | |
| | 4 | 19 | 8 | 9 | 20 | 4 | 3 | 16 | 21 | 24 | 5 | 11 | 14 | |

In this example there are no spatial neighbours, e.g. diagonal self-adjacencies, knights moves in either direction as well and other types of adjacencies as listed by Piepho, Michel and Williams (Biometrical Journal, 2018).

It also has a neighbour balance score of 0, meaning that no two treatments are neighbours more than once in either the row or column directions.

The efficiency factor for this design is 0.61 compared with 0.63 for the non-spatial design.

Spatial enhancement via NB&ED can be applied to resolvable and non-resolvable designs and also to partially-replicated (p-rep) designs. In the computer generation of designs attention should be paid not only to an optimality criterion such as A-optimality, but also to spatial criteria to ensure good NB&ED.

CycDesigN can construct efficient spatial designs for use in practice.  A new version of the package (CD8) has just been released and features a major upgrade to the generation of p-rep designs, with or without NB&ED enhancement.

P-rep block or row-column designs for single or multiple locations can include one or more standard treatments. Here is a  CD8 example of a spatial single location row-column p-rep design for 106 treatments with 34 duplicates (in bold) in a 10 x 15 array of plots.  A single standard, replicated 10 times is also incorporated in the design and spread evenly across both rows and columns.

# A spatial single location row-column p-rep design for 106 treatments with 34 duplicates

| Row | Column 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 67 | 71 | **90** | **85** | 20 | 50 | 104 | **106** | 86 | **17** | **S** | **15** | **6** | **5** | **74** |
| 2 | **46** | **91** | 2 | 69 | 38 | **48** | **S** | 73 | 34 | 57 | 1 | 99 | **77** | **7** | **84** |
| 3 | 51 | **66** | **52** | **S** | 39 | **32** | **87** | 36 | **96** | **63** | **101** | 102 | 56 | **11** | 97 |
| 4 | **S** | 10 | 12 | **15** | **37** | 80 | 72 | **31** | 88 | 14 | 100 | **16** | 82 | **61** | 26 |
| 5 | 105 | **27** | **6** | 47 | **S** | **84** | 23 | 43 | **64** | **76** | 25 | 58 | **70** | 29 | **9** |
| 6 | **18** | **63** | **17** | **101** | 3 | **5** | **11** | 30 | 68 | **S** | **40** | 28 | 79 | **91** | **85** |
| 7 | **31** | **77** | **74** | **61** | 93 | 45 | 44 | **75** | 54 | 103 | 22 | **52** | 41 | 81 | **S** |
| 8 | 33 | **96** | 8 | 59 | **70** | 95 | 19 | **S** | **46** | 89 | **27** | **37** | 53 | 78 | **106** |
| 9 | **16** | 42 | **76** | 4 | 94 | 13 | **7** | 21 | 35 | **18** | 65 | **S** | **32** | **87** | 83 |
| 10 | 60 | **64** | **40** | 98 | 24 | 62 | **9** | 55 | **S** | **66** | **90** | 49 | **48** | 92 | **75** |

There is more detail on NB&ED designs in a recent tutorial-style paper by Piepho, Williams and Michel (Journal of Agronomy, 2021) and a video on the construction of NB&ED designs using CycDesigN at Videos - CycDesigN (vsni.co.uk)

The use of spatial designs enhances the LVIS spatial analysis as described by Piepho, Boer and Williams (Biometrical Journal, 2022).